Facilitator's Guide
for
The AERO Mathematics Curriculum Framework

Acknowledgements
This guide was developed by AERO as part of the professional development opportunities to help teachers increase student achievement through the use of the AERO Mathematics Curriculum Framework.

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Introduction

Every teacher of mathematics, whether at the elementary, middle, or high school level, has an individual goal to provide students with the knowledge and understanding of the mathematics necessary to function in a world very dependent upon the application of mathematics. Instructionally, this goal translates into three components:

- conceptual understanding
- procedural fluency
- problem solving

Conceptual understanding consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. Students use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. Knowledge learned with understanding provides a foundation for remembering or reconstructing mathematical facts and methods, for solving new and unfamiliar problems, and for generating new knowledge.

Procedural fluency is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. It includes, but is not limited to, algorithms (the step-by-step routines needed to perform arithmetic operations). Although the word procedural may imply an arithmetic procedure to some, it also refers to being fluent with procedures from other branches of mathematics, such as measuring the size of an angle using a protractor. The use of calculators need not threaten the development of students' computational skills. On the contrary, calculators can enhance both understanding and computing if used properly and effectively. Accuracy and efficiency with procedures are important, but they should be developed through understanding. When students learn procedures through understanding, they are more likely to remember the procedures and less likely to make common computational errors.

Problem solving is the ability to formulate, represent, and solve mathematical problems. Problems generally fall into three types:

- one-step problems
- multi-step problems
- process problems

Most problems that students will encounter in the real world are multi-step or process problems. Solution of these problems involves the integration of conceptual understanding and procedural knowledge. Students need to have a broad range of strategies upon which to draw. Selection of a strategy for finding the solution to a problem is often the most difficult part of the solution. Therefore, mathematics instruction must include the teaching of many strategies to empower all students to become successful problem solvers. A concept or procedure in itself is not useful in problem solving unless one recognizes when and where to use it as well as when and where it does not apply. Many textbook problems are not typical of those that students will meet in real life. Therefore, students need to be able to have a general understanding of how to analyze a problem and how to choose the most useful strategy for solving the problem.

The mathematics standards, benchmarks, and performance indicators presented in this document states that students will:

- understand the concepts of and become proficient with the skills of mathematics,
- communicate and reason mathematically;
- become problem solvers by using appropriate tools and strategies;

through the integrated study of number sense and operations, algebra, geometry, measurement, and statistics and probability. Mathematics should be viewed as a whole body of knowledge, not as a set of individual components. Therefore, local mathematics curriculum, instruction, and assessment should be designed to support and sustain the components of this document.

In this document conceptual understanding, procedural fluency, and problem solving are represented as process strands and content strands. These strands help to define what students should know and be able to do as a result of their engagement in the study of mathematics.

Process Strands: The process strands (Problem Solving, Reasoning, Communication and Representation and Connections) highlight ways of acquiring and using content knowledge. These process strands help to give meaning to mathematics and help students to see mathematics as a discipline rather than a set of isolated skills. Student engagement in mathematical content is accomplished through these process strands. Students will gain a better understanding of mathematics and have longer retention of mathematical knowledge as they solve problems, reason mathematically, prove mathematical relationships, participate in mathematical discourse, make mathematical connections, and model and represent mathematical ideas in a variety of ways.

Content Strands: The content strands (Number Sense and Operations, Algebra, Geometry, Measurement, and Statistics and Probability) explicitly describe the content that students should learn. Each school's mathematics curriculum developed from these strands should include a broad range of content. This broad range of content, taught in an integrated fashion, allows students to see how various mathematics knowledge is related, not only within mathematics, but also to other disciplines and the real world as well. The performance indicators listed under each band within a strand are intended to assist teachers in determining what the outcomes of instruction should be. The instruction should engage students in the construction of this knowledge and should integrate conceptual understanding and problem solving with these performance indicators. ***The performance indicators should not be viewed as a checklist of skills void of understanding and application.***

Students will only become successful in mathematics if they see mathematics as a whole, not as isolated skills and facts. As schools develop their mathematics curriculum based upon the statements in this Curriculum Framework document, attention must be given to both content and process strands. Likewise, as teachers develop their instructional plans and their assessment techniques, they also must give attention to the integration of process and content. To do otherwise would produce students who have temporary knowledge and who are unable to apply mathematics in realistic settings. Curriculum, instruction, and assessment are intricately related and must be designed with this in mind. All three domains must address conceptual understanding, procedural fluency, and problem solving. If this is accomplished, school districts will produce students who will (1) have mathematical knowledge, (2) have an understanding of mathematical concepts, and (3) be able to apply mathematics in the solution of problems.

Schools and individual teachers should be aware that this document is a standards document that guides the development of local curriculum. In this document the mathematics standards, benchmarks, and performance indicators are succinctly stated. The standard outlines what students should know and be able to do in mathematics. The content strands and the performance indicators within each benchmark, and the performance indicators of the process strands help to define how the standard will be met. Each school mathematics curriculum should be developed to assure that all students achieve the performance indicators for both the process and content strands. Helping all students become proficient in mathematics is an imperative goal for every school. It is the hope that this document will assist schools and individual teachers in meeting this goal.

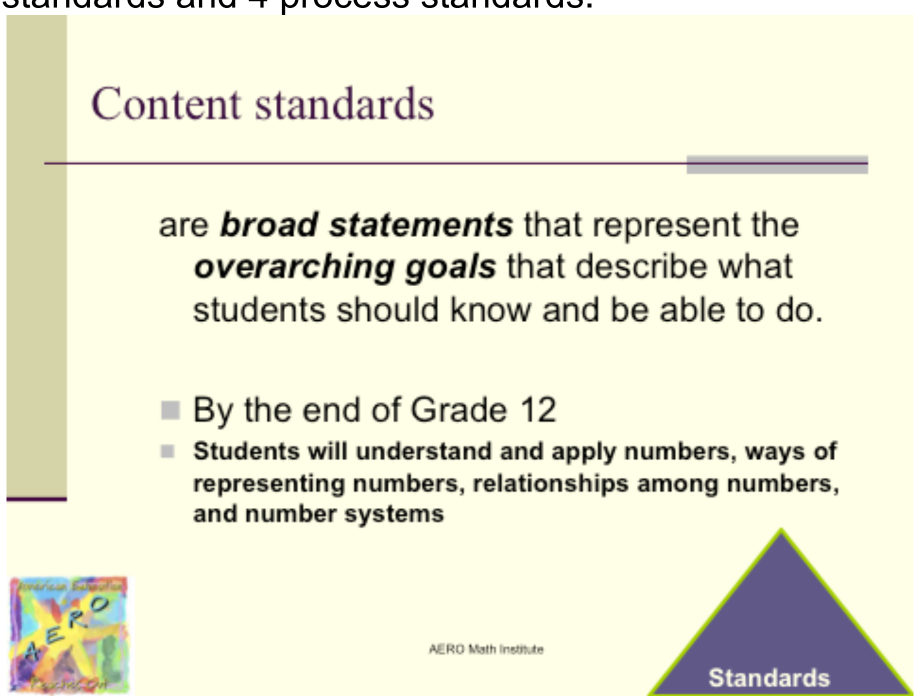
Organization of the Framework

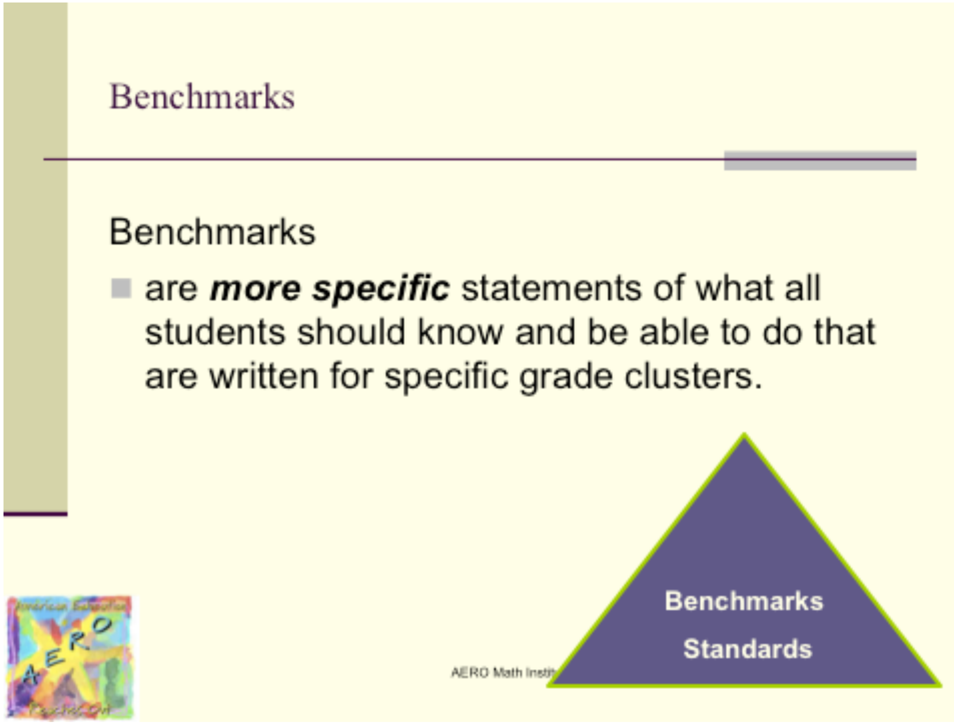
The Framework identifies specific indicators for each grade level, K-8. The standards and benchmarks reflect the National Council Teachers of Mathematics (NCTM) standards and benchmarks, the NCTM focal points, and the international benchmarking project are coordinated across grades K-8 to ensure coherence and comprehensiveness, and support focused, in-depth instruction.

(http://hrd.apec.org/index.php/Analysis_of_Mathematics_Standards#Mathematics_Standards:_A_Priority)

This document reflects the most current research and can serve as a foundation for system-wide improvement in curriculum, instruction, and assessment.

Standards are statements of what students should know and be able to do by the end of grade 12. The Framework includes 5 content standards and 4 process standards.





The Framework identifies 17 benchmarks.

For example

Standards 5.0 Numbers and Operations

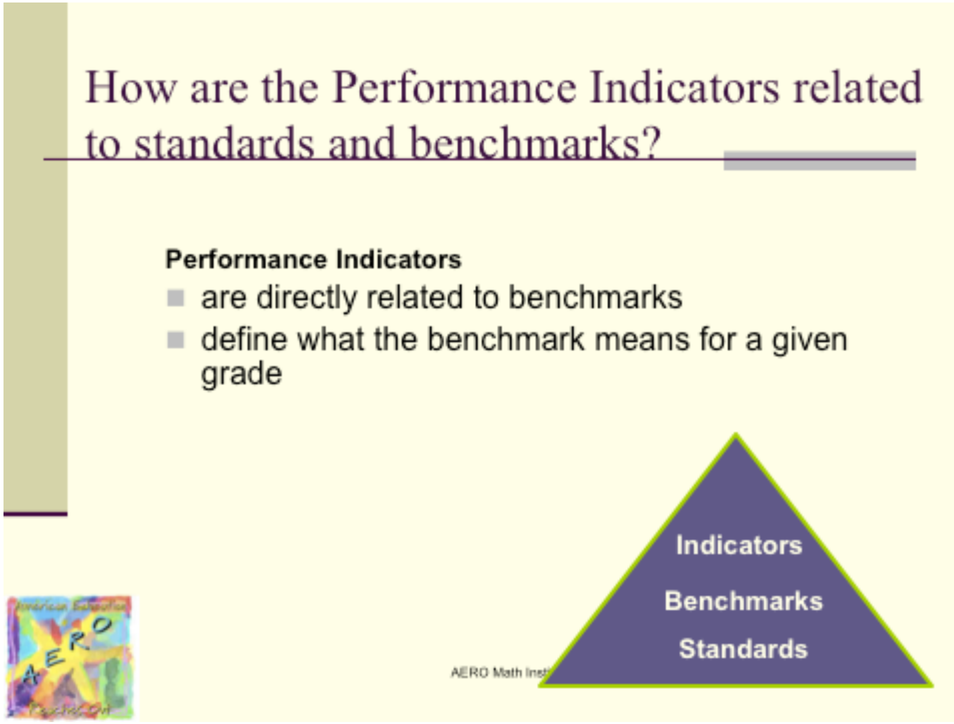
Students will understand and apply numbers, ways of representing numbers, relationships

Benchmarks

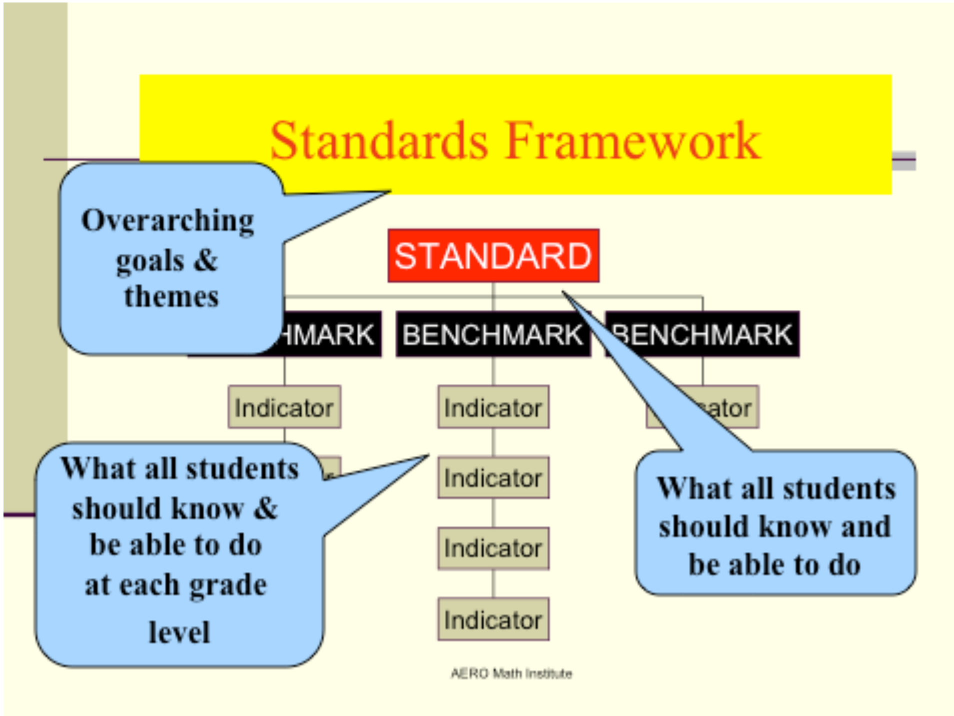
5.1 Numbers and Number Sense
Students will understand and demonstrate a sense of what numbers mean and how they are used.

5.2 Operations on Numbers
Students will understand meanings of operations and how they relate to one another.

5.3 Numerical Operations and Estimation
Students will accurately calculate and use estimation techniques, number relationships, operation rules, and algorithms; they will determine the reasonableness of answers and the accuracy of solutions.



Level	K	1	2	3	4
Counting	Count forward by 1's to 31 and backward from 10 with and without objects	Count forward by 1's to 99 and backward from 20 with and without objects	Count by tens or hundreds forward and backward starting at any number from 1 to 999 and count by twos, fives, and tens to at least 100.	Count by hundreds and thousands starting at any number from 1 to 9,999	Count by thousands and ten thousands starting at any number from 1 to 999,999

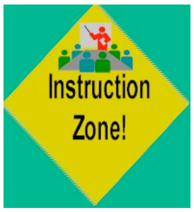


Some clarifications

Concepts:
Concepts are ideas and information that students need to know. They are listed as the nouns in the indicator statements. Identifying concepts (and skills, below) is part of a systematic process of "unpacking standards," which helps teachers to develop a deeper understanding of the indicators when planning instruction and assessment. Listing these concepts can help to develop activities and assessments that relate to the standard.

Skills:
Skills are what students are expected to do to demonstrate mastery of the concepts and content. They are listed as the verbs in the indicator statements. A single skill may apply to multiple concepts. By listing these skills, teachers begin to see and understand how they correlate to Bloom's Taxonomy of thinking skills, which teachers will use when planning performance tasks and assessments.

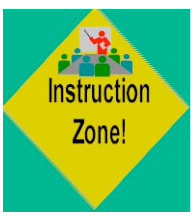
Becoming familiar with the Framework



Choose one content strand (horizontal) to analyze.

Read and analyze the strand

Summarize a definition for content strands

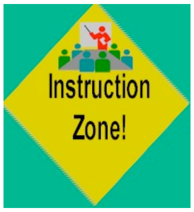


Repeat with a process strand

Read and analyze the strand

Summarize a definition for process strands.

Compare and contrast content and process strands.



Divide into small groups, each group examining a content standard

Examine the concepts

Choose one strand (horizontal) and observe when does it begin?

When does it end?

What do you notice about the grade range?

What are the implications of this for teachers? For curriculum? For assessment?

Share and discuss

Resources

The framework can be used to revise curriculum maps, parent communication and assessments. Some resources are:

STRATEGIES TO EXTEND STUDENT THINKING

Remember "wait time I and II"

Provide at least five seconds of thinking time after a question **and** after a response.

Ask "follow-ups'

E.g., "Why? How do you know? Do you agree? Will you give an example? Can you tell me more?

Cue responses to "open ended" questions

E.g., "There is not a single correct answer to this question. I want you to consider alternatives.

Use "think-pair-share"

Allow individual thinking time, discussion with a partner, and then open up for class discussion.

Call on students randomly

Avoid the pattern of only calling on those students with raised hands.

Ask students to "unpack their thinking"

E.g., ‘Describe how you arrived at your answer."

Ask for summary to promote active listening

E.g., "Could you please summarize our discussion thus far?"

Play devil's advocate

Require students to defend their reasoning against different points of view.

Survey the class

E.g., "How many people agree with the authors point of view?" (thumbs up, thumbs down)

Allow for student calling

E.g., "Richard, will you please call on someone to respond?"

Encourage student questioning

Provide opportunities for students to generate their own questions.

Developing Mathematical Thinking with Effective Questions

To help students build confidence and rely on their own understanding, ask…

- Why is that true?
- How did you reach that conclusion?
- Does that make sense?
- Can you make a model to show that?

To help students learn to reason mathematically, ask…

- Is that true for all cases? Explain.
- Can you think of a counterexample?
- How would you prove that?
- What assumptions are you making?

To check student progress, ask…

- Can you explain what you have done so far? What else is there to do?
- Whey did you decide to use this method?
- Can you think of another method that might have worked?
- Is there a more efficient strategy?
- What do you notice when…?
- Why did you decide to organize your results like that?
- Do you think this would work with other numbers?
- Have you thought of all the possibilities? How can you be sure?

To help students collectively make sense of mathematics, ask…

- What do you think about what _____ said?
- Do you agree? Why or why not?
- Does anyone have the same answer but a different way to explain it?
- Do you understand what _____ is saying?
- Can you convince the rest of us that your answer makes sense?

To encourage conjecturing, ask…

- What would happen if…? What if not?
- Do you see a pattern? Can you explain the pattern?
- What are some possibilities here?
- Can you predict the next one? What about the last one?
- What decision do you think he/she should make?

To promote problem solving, ask…

- What do you need to find out?
- What information do you have?
- What strategies are you going to use?
- Will you do it mentally? With pencil and paper? Using a number line?
- Will a calculator help?
- What tools will you need?
- What do you think the answer or result will be?

To promote problem solving, ask…

- What do you need to find out?
- What information do you have?
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- Will a calculator help?
- What tools will you need?
- What do you think the answer or result will be?

To help when students get stuck, ask…

- How would you describe the problem in your own words?
- What do you know that is not stated in the problem?
- What facts do you have?
- How did you tackle similar problems?
- Could you try it with simpler numbers? Fewer numbers? Using a number line?
- What about putting things in order?
- Would it help to create a diagram? Make a table? Draw a picture?
- Can you guess and check?
- Have you compared your work with anyone else? What did other members of your group try?

To make connections among ideas and applications, ask…

- How does this relate to…?
- What ideas that we have learned before were useful in solving this problem?
- What uses of mathematics did you find in the newspaper last night?
- Can you give me an example of…?

To encourage reflection, ask…

- How did you get your answer?
- Does you answer seem reasonable? Why or why not?
- Can you describe your method to us all? Can you explaining why it works?
- What if you had started with… rather than…?
- What if you could only use…?
- What have you learned or found out today?
- Did you use or learn any new words today? What did they mean? How do you spell them?
- What are the key points or big ideas in this lesson?

COMPLEXITY OF TASKS

National Assessment of Educational Progress – Mathematical Complexity

Low Complexity

This category relies heavily on the recall and recognition of previously learned concepts and principles. Items typically specify what the student is to do, which is often to carry out some procedure that can be performed mechanically. It is not left to the student to come up with an original method or solution. The following are some, but not all, of the demands that items in the low-complexity category might make:

- Recall or recognize a fact, term, or property.
- Recognize an example of a concept.
- Compute a sum, difference, product, or quotient.
- Recognize an equivalent representation.
- Perform a specified procedure.
- Evaluate an expression in an equation or formula for a given variable.
- Solve a one-step word problem.
- Draw or measure simple geometric figures.
- Retrieve information from a graph, table, or figure.

Moderate Complexity

Items in the moderate-complexity category involve more flexibility of thinking and choice among alternatives than do those in the low-complexity category. They require a response that goes beyond the habitual, is not specified, and ordinarily has more than a single step. The student is expected to decide what to do, using informal methods of reasoning and problem solving strategies, and to bring together skill and knowledge from various domains. The following illustrate some of the demands that items of moderate complexity might make:

- Represent a situation mathematically in more than one way.
- Select and use different representations, depending on situation and purpose.
- Solve a word problem requiring multiple steps.
- Compare figures or statements.
- Provide a justification for steps in a solution process.
- Interpret a visual representation.
- Extend a pattern.
- Retrieve information from a graph, table, or figure and use it to solve a problem requiring multiple steps.
- Formulate a routine problem, given data and conditions.
- Interpret a simple argument.

High Complexity
High-complexity items make heavy demands on students, who must engage in more abstract reasoning, planning, analysis, judgment, and creative thought. A satisfactory response to the item requires that the student think in abstract and sophisticated ways. Items at the level of high complexity may ask the student to do any of the following:

- Describe how different representations can be used for different purposes.
- Perform a procedure having multiple steps and multiple decision points.
- Analyze similarities and differences between procedures and concepts.
- Generalize a pattern.
- Formulate an original problem, given a situation.
- Solve a novel problem.
- Solve a problem in more than one way.
- Explain and justify a solution to a problem.
- Describe, compare, and contrast solution methods.
- Formulate a mathematical model for a complex situation.
- Analyze the assumptions made in a mathematical model.
- Analyze or produce a deductive argument.
- Provide a mathematical justification.

Diagnosing learning difficulties in math

To diagnose math difficulties, look at

- What evidence is there of a math difficulty ?
- What math does the student know?
- How can the math disability be explained ?

<u>Step 1 Describe math performance</u>

Collect samples of math tasks for analysis, keep a record of

- the tasks,
- the conditions
- the learner's output,
- the time taken.
- what the learner says as they work through the tasks.
- note learners' level of stress and any behavioral indicators of this.
- note how math ability changes when learners are cued to operate in a particular way.

<u>Step 2 Analyzing math performance.</u>

To complete any math task, pupils need to
• think about the ideas in particular ways; to visualize and say them, model them link them with what is known, question them. These are the task processing strategies.

• manage the math activity; they need to

- plan how to manipulate the data, select the aspects of their knowledge they will use
- decide when to paraphrase, visualize or represent concretely
- decide when to pause and consolidate,
- check that they are on the right track or take remedial action.

To complete any task, they use these strategies in the following order: they

• inform themselves of the task, that is, read the data defining the task. They must
  - comprehend the meaning of each element,
  - integrate or combine the meanings in the intended ways,
  - discriminate between relevant and irrelevant data,
  - decide what the completed task will be like
  - link the task to types of tasks learned previously; they categorize it as an instance of types already learned,
  - recall and apply appropriate procedures to the data given,
  - recall particular number facts,
  - monitor the effectiveness of their efforts, and if these are judged to have been unsuccessful, to rework the task.

The purpose is to identify the learning strategies students use while doing tasks. Ask students to work through tasks aloud or to report how they went about doing them ('reflective mathematics assessment).

Note whether they
  - use various strategies independently,
  - know when to use each strategy,
  - can apply the strategy to a range of instances or to simpler examples.

Step 4 Diagnosing Student Errors

	TASK 1	TASK 2	TASK 3
task orienting stage			
say each symbol or element and what it means			
describe the task, perhaps drawing or making it concretely			
say what the outcome will be like			
say how the task is similar to types of tasks learned and say how they decided			
say what they will do first, second, etc			
'implementing stage			
apply the steps in an integrated, systematic way			
monitor how they are doing the task, whether they are getting closer to a solution			
recall relevant number facts; note how automatically this is done.			

review, consolidating stage			
decide whether the outcome is reasonable, possibly correct			
review what they have done in terms of its adequacy and if necessary take remedial action			